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Themen, Methoden, Perspektiven

herausgegeben von Martin Pfleiderer
und Wolf-Georg Zaddach

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Constructing Jazz Lines. Taxonomy, Vocabulary, Grammar

Abstract (English)

In this paper, we propose a novel classification system for improvised jazz lines, which we have called the Weimar Bebop Alphabet. It is based on a phrase-wise parsing of interval sequences using nine different classes of melodic atoms (diatonic, chromatic, approaches, arpeggios, jump arpeggios, repetitions, trills, links and a residual category called X atoms). The system is applied to 456 solos from the Weimar Jazz Database and basic statistical properties are reported. The mean length of atoms is rather short, with 2.4 tones. The most common approaches, arpeggios, links and X atoms are discussed in more detail. Finally, first order Markov models of melodic atoms are investigated and the resulting findings show that the succession of atoms is close to pure randomness. This reflects the high variability and complexity of jazz melodies and runs counter to the inital assumption of a substantially simplified description and a structure-rich grammar for melodic construction. Ideas for the future extension, refinement and application of the proposed classification system are presented.

Abstract (Deutsch)

Wir stellen ein neues Klassifikationssystem für improvisierte Jazzlinien vor – das Weimar Bebop Alphabet. Es besteht aus einer Zerlegung von Intervallfolgen mit Hilfe von neun verschiedenen Klassen melodischer Atome (diatonische/chromatische

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Skalenausschnitte, Approaches, Arpeggios, Sprungarpeggien, Tonwiederholungen, Triller, Verbindungsglieder (Links) und eine Restkategorie genannt X Atome. Das System wurde auf die 456 Soli in der Weimar Jazz Database angewandt, wobei die Zerlegung phrasenweise erfolgt. Wir berichten einige grundlegende statische Kennwerte und Verteilungen, z.B. ist die mittlere Länge der Atome mit 2,4 Tönen eher kurz. Die häufigsten Approaches, Arpeggien, Links und X Atome sind von besonderem Interesse und werden etwas genauer beschrieben und diskutiert. Weiterhin werden Markow-Modelle erster Ordnung untersucht, wobei die Ergebnisse zeigen, dass die Abfolge der melodischen Atome fast komplett zufällig ist. Dies spiegelt zwar die große Variabilität und Komplexität improvisierter Melodien im Jazz wider, läuft aber auch etwas der anfänglichen Idee zuwider, mit dem System eine substantiell vereinfachende Darstellung mit einer strukturreichen Grammatik für Jazzmelodien zu entwickeln. Schließlich werden noch Ideen für zukünftige Verfeinerungen, Erweiterungen und Anwendungsfälle des hier vorgeschlagenen Klassifikationssystem vorgestellt.

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Constructing Jazz Lines
Taxonomy, Vocabulary, Grammar

Introduction

Inarguably, long, angular, and meandering lines are one of the defining features of jazz solo improvisation. Presumably, they achieved their current prominent role historically with the advent of bebop (Frieler 2018). In bebop and most of the following jazz styles, displaying virtuosity by playing blazingly fast (bebop) lines with many twists and turns while still outlining the harmonies (›running the changes‹) is a skill that demands extensive practice and also a certain amount of intellectual and cognitive abilities. Bebop lines can be viewed as the endpoint of a longer trend in jazz improvisation, where the lengths and boundaries of shorter licks and phrases, which were common in earlier improvisations of traditional and swing jazz, were increasingly pushed to further extremes in a kind of virtuosic arms race (DeVeaux 1997; Frieler 2018; Owens 1995). Lester Young, Coleman Hawkins and Benny Goodman, amongst others, can be seen as important improvisers who were already moving in this direction during the 1930s. The ›new cats‹ of the emerging bebop style, e.g., Charlie Parker, Dizzy Gillespie, Fats Navarro, Sonny Stitt, and many others, made fast tempos the norm instead of the exception. In this context, they developed the art of playing very fast lines, which since then has become a staple of jazz improvisation. A rather extreme example of a very long bebop line with 80 notes, played by Fats Navarro in 1948, is depicted in Fig. 1.

Jazz lines, as a musical but also as a psychological and socio-cultural phenomenon, have received little systematic treatment from the jazz research community. The emphasis here is on ›systematic‹, since a large part of classical jazz analysis literature (e.g., Berliner 1994; DeVeaux 1997; Kerschbaumer 1978; Owens 1995; Waters 2011, to name but a few) deals with constructive principles in jazz improvisation, often with a focus on pitch choices with respect to the underlying harmonies. Melodic elements such as scales, arpeggios, approaches (enclosures), octave

Fig. 1: Example of a very long bebop line as played by Fats Navarro in his solo on »Good Bait« (1948, Trumpets of Jericho Ltd. 20.1976-HI), transcription taken from the WJD. The actual line as annotated in the WJD starts on the second triplet in m. 16 and ends on the second eighth in m. 21, comprising a staggering total of 80 tones.

displacement etc. are often used in jazz analyses and are also omnipresent in the vast body of textbooks on jazz improvisation (Aebersold 1967; Baker 1988; Coker 1987), academic courses, and YouTube instructional videos[1]. However, no comprehensive and unequivocal classification system for the basic building blocks in monophonic jazz improvisations has been developed so far. The present study is an attempt to fill this gap with a novel approach tentatively called the *Weimar Bebop Alphabet* (WBA). It provides a reduction of the melodic surface by devising a classification system of melodic ›atoms‹, which are situated between the tone level and the level of midlevel units. The first benefit of such a system is that it can be implemented in a computer program and thus facilitate the statistical analysis of a large corpus of jazz solos in order to create a phenomenology of melodic elements.

The ultimate goal is, however, a hierarchical model of jazz improvisation, where midlevel units (MLU) are realized with sequences of WBA atoms, which in turn are realized as sequences of tone events over a given chord sequence. Midlevel units are defined by midlevel analysis (MLA), an annotation system designed for categorizing ›playing ideas‹ in jazz solos (Frieler/Pfleiderer/Abeßer/Zaddach 2016). According to this model, WBA atoms are small chunks of well-rehearsed melodic movements

1 A quick Google search for the terms »jazz improvisation + X« yielded about 3 mio. results for X = »scale«, about 1 mio. for X = »arpeggio«, and about 300,000 for X = »enclosure«.

which presumably reside mostly in motor memory. A performer can easily adapt (or select) WBA atoms according to the local harmony and chain them to larger units. They thus, we argue, play an active, productive role in the improvisation process. Therefore, the WBA system is guided by certain assumptions about actual mental production processes during jazz improvisation, but also provides a neutral, purely descriptive system for melodies.

Background and related work

The concept of melodic reduction is far from new, and many formal and semi-formal systems have been proposed. Some important approaches are based on linguistic or Schenkerian ideas (Larson 1987; Lerdahl/Jackendoff 1983; Nattiez 1975; Roads 1979; Schenker 1956), and attempt to find grammars for melodies (or other elements of music). These are often context-free grammars, which consist of production rules over an ›alphabet‹ and define how elements (non-terminal symbols) can be recursively substituted for other elements. This can be used for analysis and generation alike. For example, ›The old man plays the blues‹ can be parsed into a subject (›The old man‹), a predicate (›plays‹), and an object (›the blues‹), and the subject can then be further parsed into an article (›the‹), an adjective (›old‹), and a noun (›man‹). One example of a melodic production rule would be repeating a note and inserting an upper neighboring tone between the repetitions, which results in a mordent (Gilbert/Conklin 2007). Musical grammars proposed in the literature pursue many goals, e.g., tonal analysis (Lerdahl/Jackendoff 1983), predicting phrases in folk songs (Bod 2002), estimation of information content and compressibility (Gilbert/Conklin 2007), and music generation (Cope 2005; Johnson-Laird 1991; Keller/Morrison 2007; Roads/Strawn 1985; Steedman 1984). Grammar-based reductions of the melodic surface suffer from the segmentation problem, i.e., the fact that there are no natural units in music that might play a role analogous to that of words in a language. Hence, it is not clear a priori which consecutive chunks of tones form a unit that serves as a basic element in the next hierarchy level. In natural language, this problem is not prevalent. For instance, ›the old man‹ serves unequivocally as the subject in the overall sentence structure SPO. Parsing music is likewise hampered by the fact that there are no predefined semantic functions or roles for musical events that might guide the parsing process. There is no equivalent to a noun or an adjective in music, nor is there defined representational meaning in music. Therefore, either external, auxiliary criteria are needed to build a grammar, or it has to be done in a purely data-driven fashion (Bod 2002). Furthermore, we claim that melodic grammars in music have only few, if any, levels of recursion. This is in line with Johnson-Laird's principle of »algorithmic demands« (Johnson-Laird 2002), which states that jazz improvisation does not need working memory (even though we would like to stress that this can only be approximately true). In essence, these ›flat‹ grammars are very much like Markov models.

Markov models have also been applied to music analysis and generation for a long time. They assert that the probability of the next event depends only on a certain (short) history of past events and thus capture some form of a ›local‹ grammar in stating rules about which elements are likely to follow each other. Markov models do not feature the hierarchical and recursive aspect of grammars, but can be combined with them to form hybrid models. Applications of Markov models to music are manifold and serve a range of different purposes, e.g., music generation (Cope 2005; Hiller/Isaacson 1959), modeling melodic expectations (Pearce 2005), folk song classification (Chai/Vercoe 2001), and music information retrieval in general (Typke/Wiering/Veltkamp 2005).

Another important system of melodic reduction which shares some similarities with the WBA system is the Implication-Realization (I/R) model formulated by Narmour (1990), which is based on Gestalt principles and models melodic expectations (›implications‹) using five different grouping principles, e.g., »registral direction«, stating that small intervals (less than 6 semitones) imply a continuation in the same direction, whereas larger intervals imply a change of direction (»registral return«). Whether an implication is realized or not permits a classification of interval pairs, which can be used to parse a melody into I/R units (Grachten/Arcos/López de Mántaras 2005).

The Weimar Bebop Alphabet

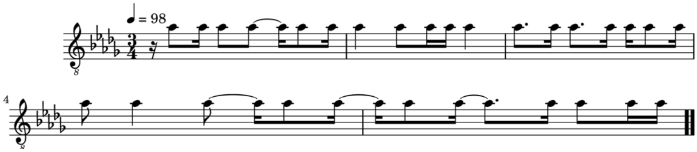
Overview

The main idea of the proposed approach is to group chunks of consecutive tones into a few distinct classes. Rhythmic and metrical information will be ignored for this purpose to avoid complicating the matter further. Likewise, absolute pitch will be disregarded in favor of using semitone intervals to achieve transposition invariance.

Starting from a melodic representation of pitch, onset and duration (p_i, t_i, d_i), we first drop all timing information, resulting in pitch sequences p_i, which are then transformed into interval sequences $\Delta p_i = p_{i+1} - p_i$, which have one fewer element than the original sequences. Next, a set of nine distinct classes which will allow an exhaustive and unique classification of melodic chunks is used. The classes are devised based on commonly stated melodic elements in jazz and music theory and are the following.

1. Repetition (R) is the simplest way to construct melodies and can be found in all music. A repetition atom needs at least two intervals (three tones). An example can be found in Fig. 2a.
2. Diatonic and chromatic scale extracts (D, C) are ubiquitous in music. It is known that whole-tone and semitone steps are the most common intervals not only in jazz, but in nearly all of music (Frieler et al. in press). D and C atoms need to have at least two intervals (three pitches) of the same direction (ascending or descending). See Figs. 2b and 2c for examples.
3. Arpeggios (A, J). Western harmony is based on the vertical stacking of thirds, which, when linearized, results in arpeggios. Arpeggios are horizontal versions of vertical harmony and are often used as such, e.g., in accompaniment figures. Since chorus-based jazz improvisation is mainly based on the underlying harmonies, arpeggios provide a simple and effective means to convey, extend, complement, or contrast these harmonies. Using chord inversions, arpeggios can also consist of intervals other than (mostly larger than) thirds, which results in the jump arpeggios subtype (J). Arpeggios need to have a least two intervals (three pitches) in the same direction. See Figs. 2d and 2e for examples.
4. Trills (T) are very common embellishments in Western music (also comprising mordents). They can be viewed as a special case of tone repetitions, where a group of tones is repeated instead of a single tone. In agreement with classical ornament theory (e.g., Neumann and Bach 1983), we restrict ourselves to trills with two repeating pitches a semitone or a whole tone apart. Trills need to have at least two intervals (three pitches). See Fig. 2f for an example.

a



b

c

d

e

f

g

h

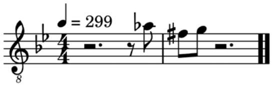


Fig. 2: Examples of WBA atoms taken from the WJD. (a) Repetition (=R24), (b) ascending chromatic atom (+C15), (c) descending diatonic atom (-D15), (d) simple ascending arpeggio (+A7), (e) extended descending arpeggio (–D7), (f) whole tone trill (=T23), (g) X atom (–X21), (h) [–2,1] approach (–F). Note: For the purposes of demonstration, unusually long examples were selected (cf. Fig. 4A). Solo sources: (a) Steve Coleman, »The Oracle«; (b) Phil Woods, »Strolling with Pam«; (c) Dave Liebman, »Day & Nite«; (d) George Coleman, »Maiden Voyage«; (e) Eric Dolphy, »Serene«; (f) Wayne Shorter, »Juju«; (g) Sonny Rollins, »The Everywhere Calypso (2)«.

5. Approaches (F) are a rather jazz-specific figuration, though they can appear in any melody. Approaches are one of the few elements of bebop line constructions that are commonly mentioned and taught (e.g., Kissenbeck 2007). They are defined here as chunks of two intervals (three pitches) that have (1) a change of direction, (2) a net movement of at most a whole tone, (3) a maximum interval size of a major third (±4). We also stipulate that at least one interval is a whole tone or a semitone. This gives a total of eight possible approaches, four ascending and four descending each for semitone and whole-tone net movements. Approaches can be further classified as two different types: ›enclosures‹, where the target tone is between the framing tones, and ›escaping‹ approaches, where the third tone is outside the range of the first two, such as in [–1, 3]. Enclosures are characterized by the property of the first interval being absolutely larger than the second. As we will see later, intrinsically chromatic approaches are of special interest in jazz improvisation. These are the two enclosures [2, –1] and [–2, 1] and the two escaping approaches [1, –2] and [–1, 2]. They comprise a pitch set of three semitones and are hence not a subset of a diatonic scale. See Fig. 2h for an example of a chromatic approach.
6. X atoms and links (X, L). These are residual categories which comprise all melodic chunks that cannot be classified in one of the other categories. While residual atoms with two or more intervals are labelled ›X atom‹, residual atoms of only one interval are called links (L), to indicate their supposed function of linking two atoms. See Fig. 2g for an example of a rather long X atom.

We argue that this system captures important principles of (Western) diatonic melody construction and reflects musical thinking on the part of the improvisors, at least to some extent. Of course, the current system is only preliminary and future refinements and revisions might be deemed necessary after an analytical application.

A design decision had to be made as to whether or not to allow overlap between atoms. For example, consider a case like [–1, –1, 1, 2, 2, –1]. The first two intervals [–1, –1] constitute a descending chromatic atom, whereas the next two intervals [–1, 1] can be interpreted as a semitone trill, while the next three intervals [1, 2, 2] form an ascending diatonic atom. Finally, the last two intervals [2, –1] are an ascending chromatic approach.

So each atom, according to the basic definition, overlaps the next one by one element. There are good arguments both for allowing and for disallowing overlaps. Using overlaps might in some way be more appropriate to the actual production process while also stressing the interlocking of atoms. On the other hand, overlaps would complicate statistical analysis because assumptions of statistical independence are immediately violated, and calculating Markov transitions probability is then no longer feasible. For this reason, we finally decided to exclude overlaps. However, to disambiguate multiple possible interpretations of an interval sequence, a priority list has to be introduced for parsing the atoms. The general algorithm proceeds by finding atoms of maximal length according to the following priorities: (1) repetitions, (2) scales, (3) arpeggios, (4) trills, (5) approaches, and (6) X atoms and links.

Another important ingredient of the WBA algorithm is the principle of ›maximal length in one direction‹ for diatonic, chromatic, and (jump) arpeggios. This means that the length of each atom is defined by the longest sequence in the same direction (ascending or descending). For example, the interval sequence [1, 2, 2, −2, −2, −1] will be parsed into one ascending diatonic atom [1, 2, 2] followed by one descending chromatic atom [−2, −2, −1]. This principle, in conjunction with the priority list, allows a unique parsing of the interval sequences into non-overlapping WBA atoms. Applying the principle can be justified by the principle of »registral direction« from Narmour's I/R model (Narmour 1990), where the D, C, and A atoms can be viewed as a realization of this implication. This is not always true for J atoms, which might have larger interval sizes as required by the I/R model, according to which they can be considered a special case of arpeggios. Likewise, repetition atoms are also realizations of this principle with the smallest possible interval, the unison. On the other hand, approaches (and formally also trills, although they follow a different logic) can be regarded as violations (non-realizations) of this principle as they involve small intervals but also a change of direction. Moreover, all these atoms are realizations of the principle of »interval difference« from the I/R model, which states that small intervals imply a continuation with intervals of similar size.

Atoms are thus characterized by two basic properties: overall direction (ascending, descending, and static) and length (number of intervals). The direction of an interval pattern is defined by the sign of the sum of its constituent intervals, e.g., the X atom [−2, −5, 3, 2, −5, 0] with five elements has a sum of –7 indicating a descending direction. We introduced a short notation of the form *[direction][type][length]*, where *direction* is one of +, – or =, *type* is a single capital letter as defined in the list above, and *length* is the number of elements (intervals) in the atom. For example, said X atom would be denoted as –X6, and a sevenfold repetition would be written as =R7 (cf. Fig. 2 for more examples). The first three bars of Fats Navarro's line on »Good Bait« with complete WBA annotation can be found in Fig. 3.

Method

Data

The Weimar Jazz Database contains high-quality transcriptions of 456 monophonic solos by 78 different soloists with over 200,000 tone events. A solo transcription is represented as a list of tone events with the three core parameters *onset* (in seconds), *duration* (sec) and *pitch* (in MIDI numbers, 0–127, C4 = 60, representing semitones) and a vast array of annotations such as metrical position, intensity, and f0-modulations (vibrato, glides, slides, etc.), as well as segmental annotations such as phrase and chord contexts, chorus IDs, and mid-level units. Transcriptions and annotations were produced manually by expert transcribers and carefully double-checked. An overview of the data set can be found in Table 1 and a full list of solos is available on the Jazzomat website.[2]

Procedure

Using the algorithm as outlined above, all solos were automatically annotated with sequences of WBA atoms based on a pre-segmentation into phrases (as annotated in the WJD) and additionally based on a segmentation into midlevel units (Frieler et al. 2016). Descriptive statistics and the first Markov transition matrix were calculated for further analysis.

2 http://jazzomat.hfm-weimar.de/dbformat/dbcontent.html.

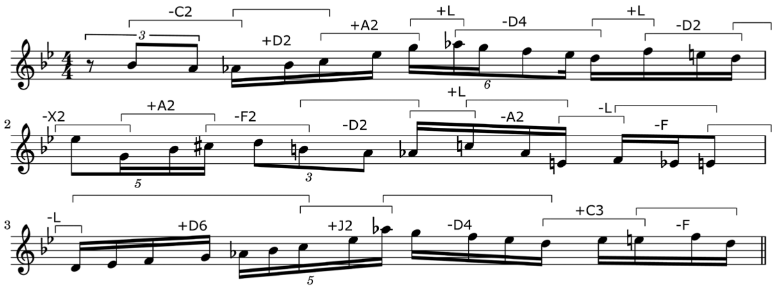


Fig. 3: The first three bars of the Fats Navarro line over »Good Bait« from Fig. 1 with complete WBA annotation.

Solos	456
Performers	78
Number of Tones	200,809
Performers with the most solos	Coltrane (20), Davis (19), Parker (17), Rollins (13), Liebman (11), Brecker (10), Shorter (10), S. Coleman (10)
Styles	Traditional (32), swing (66), bebop (56), cool (54), hardbop (76), postbop (147), free (5 = O. Coleman)
Instruments	ts (158), tp (101), as (80), tb (26), ss (23), other (68)
Time range	1925–2009

Table 1: Overview of the Weimar Jazz Database.

Results

General statistics

In the Weimar Jazz Database, *lines* comprise about 31.5% of all midlevel units and account on average for 40.2% of the total duration of a jazz solo. The second most common category, *licks*, comprises 45.7% of all MLUs, but only 36.9% of the total duration (Frieler et al. 2016). This is due to the higher number of notes in a *line*, which is 19.4 notes on the average, compared to an average of 8.3 notes for *lick* MLUs. Among the 18 different subtypes of lines, ›wavy‹ lines, representing the typical bebop lines with their many twists and turns, are the most common, accounting for 18.6% of all MLUs and for 29.1% of the total duration.

The parsing algorithm yielded 80.600 WBA atoms in total. Absolute and relative frequencies of atoms with respect to type can be found in Table 1. The most common atoms are the diatonic (D), the link (L), and the X atom with about 20% each. The next most common atoms are approaches (F), simple arpeggios (A), and chromatic atoms (C), with about 10% each. The least common are jump arpeggios (J), trills (T), and repetitions (R) with about 5% each. Recalling that links are a subgroup of X atoms, they jointly account for about 40% of all atoms. This is an unanticipated result that reflects the high variability and unpredictability of jazz solos.

MLUs contain 5.21 atoms on average and the atoms have a mean length of 2.49 tones. Even disregarding approaches and link atoms, which have fixed lengths of two and one tones, respectively, the average number of tones per atom is still only 2.74. The distribution of atom lengths can be seen in Fig. 4A. All distributions are of power-law type with a maximum at the minimal length of two intervals and with very short tails except for the X, D, R, and T atoms. This is also an interesting result, as it shows that atoms are rather short and quickly changing.

WBA Atom	Count	Rel. Freq. (%)
Diatonic (D)	16750	20.8
Link (L)	16564	20.6
X atom (X)	14888	18.5
Approach (F)	7897	9.8
Chromatic (C)	7409	9.2
Arpeggio (A)	6603	8.2
Jump Arpeggio (J)	5101	6.3
Trill (T)	4156	5.2
Repetition (R)	1232	1.5

Table 2: Basic count and relative frequencies of WBA atoms in the WJD.

Distributions of atom directions are shown in Fig. 4B. Ascending and descending directions are roughly equally common for most atoms, with a slight tendency to descending motion, corresponding to the overall trend of descending intervals in the WJD. Approaches show a rather strong tendency to descending directions (64% vs. 35%). Likewise, the descending tendencies of chromatic atoms are also rather strong (60% vs. 40%) and increase with the length of the C atom (e.g., 70% descending vs 30% ascending C atoms of length 5). Note the rather large share (11%) of X atoms with no net movement. This subclass contains symmetrical patterns, e.g., [7, –6, 6, –7], asymmetric figurations, e.g., [7, –2, –5], and trills with larger intervals, e.g., [7, –7, 7, –7] (cf. Section 4.2.4 below).

The distribution of atoms with respect to MLU type is shown in Fig. 5. We see slightly different distributions ($x^2(56) = 4993.3$, $p < 2.2 \cdot 10^{-16}$, Cramer's V = .094). For *line* MLUs, diatonic atoms are the most frequent, whereas they are only the second most frequent atom overall (with links being the most common). Moreover, *line* MLUs contain proportionally more arpeggios and chromatic atoms. In contrast, repetitions are almost completely absent from lines and trills, and X atoms are also less frequent. *Lick* MLUs on the other hand have more X, L, and R atoms compared to the overall distribution. Interestingly, even though *rhythm* MLUs have more repetition and trills as compared to the population, these are not the most common atoms, which are still L, D, and X atoms. This is due to the fact that many *rhythm* MLUs are actually built from repeated (diatonic) multi-tone motifs, e.g., [–9, 7, 2, –9, 7, 2, –9, 7, 2, –9, 7, 2, –9] in John Coltrane's solo in »Impression« from 1963 (m. 396).

Specific atoms

In this section, we would like to discuss the properties and value distributions of some selected classes, specifically, simple arpeggios, approaches, links, and X atoms, as these can have interesting substructures. Repetition, diatonic, chromatic, and trill atoms are very narrowly defined and jump arpeggios are rather rare, so we have refrained from further analysis of these types.

Arpeggios

Besides diatonic and chromatic scales, arpeggios are commonly used in practicing regimes, and might, as a result, be very well-rehearsed by most

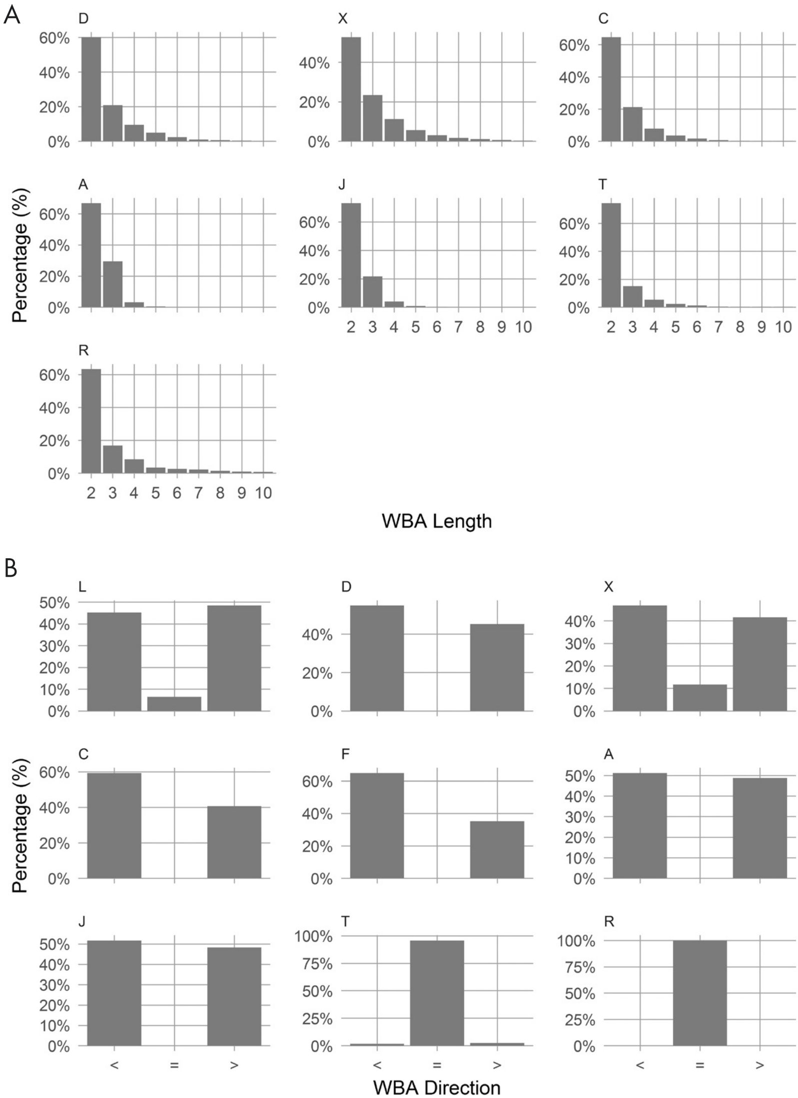


Fig. 4: Distribution of atom lengths and direction sorted by type. (A) Length distribution. Approaches and links are left out because of their fixed length. Maximum displayed length is 10. (B) Direction distribution. Repetition atoms are left out because of their constant direction.

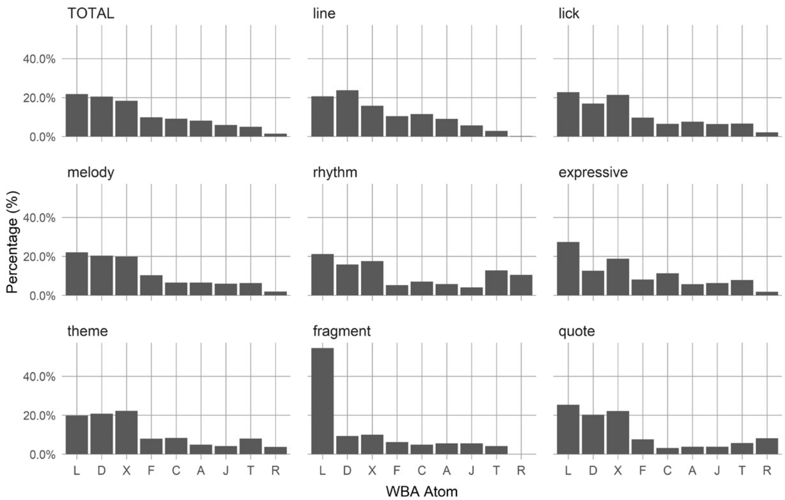


Fig. 5: Distribution of atoms with respect to MLU type. The panels are sorted from left to right and top to bottom by relative length (number of atoms) of MLUs; atom types (x-axis) are sorted for relative overall frequency. The upper right panel TOTAL is the overall distribution. For a detailed explanation of MLU types, please see Frieler et al. (2016).

players. Furthermore, arpeggios are outlining chords and are thus ideal for conveying harmony and tonality in a line. Most of the arpeggios in the WJD are rather short: 62% are triads (two intervals), 33% are 4-note chords (three intervals), and only 5% have four or more intervals. A complete breakdown of triads and 4-note chords with respect to interval content, direction, harmonic content and relative frequency can be found in Table 3.

The top five triads (ascending and descending minor and major triads as well as the ascending diminished triad) are roughly equally common. Except for the diminished triad, the descending versions are slightly more frequent than the ascending ones, in line with a general preference for descending movement. For the 4-note chords, descending and ascending minor seventh chords are far the most frequent with a total of about 34%. In contrast to the triads, ascending 4-note chords are slightly more common than descending ones, besides the fact that the descending minor seventh chord is the most frequent. Given that dominant seventh chords are the most

Intervals	Direction/ Type	Most Common Start CPC	Rel. Freq (%)
[−3, −4]	↓min	7, 10, 0	19.8
[−4, −3]	↓maj	7, 2, 4	18.4
[3, 4]	↑min	7, 0, 2	15.4
[3, 3]	↑dim	4, 9, 7	14.0
[4, 3]	↑maj	0, 3, 7, 10	13.8
[−3, −3]	↓dim	10, 3, 1	10.0
[−4, −4]	↓aug	8, 7	5.3
[4, 4]	↑aug	4, 0	3.4
[−3, −4, −3]	↓m7	10, 7, 5	21.3
[3, 4, 3]	↑m7	0, 4, 7	15.1
[3, 3, 3]	↑o7	4, 9	10.0
[4, 3, 4]	↑maj7	3, 0, 10	8.7
[3, 3, 4]	↑m7b5	4, 9, 11	7.9
[−4, −3, −3]	↓m7b5	7, 2, 8	6.2
[−4, −3, −4]	↓maj7	2, 7, 9	5.7
[4, 3, 3]	↑dom7	0, 3, 10	4.3
[−3, −3, −4]	↓dom7	10, 3	4.1

Table 3: List of simple arpeggios with 2 or 3 elements (triads and 4-note chords). Relative frequencies are given with respect to number of elements. For tetrads, only values with relative frequency greater than 4% are shown. In the column direction/ type, ↓ indicates a descending and ↑ an ascending arpeggio. Type is given using the standard jazz chord notation (maj = major triad, min = minor triad, dim = diminished triad, aug = augmented triad, maj7 = major seventh chord, m7 = minor seventh cord, o7 = (full) diminished seventh chord, m7b5 = half diminished seventh chord, dom7 = dominant seventh chord). Most common CPC indicates the three most common starting chordal pitch classes for this arpeggio, where the numbers (0–11) represent the chromatic pitch class with respect to the root of the underlying chord.

common chord type in the WJD, dominant seventh arpeggios are rather rare (about 4% each for the ascending and descending versions). For triads, a tendency to start either on the root or the fifth (CPC classes 0 and 7, cf. Table 3) can be seen. Interestingly, the descending augmented triad starts not only on the minor sixth (b13, CPC 8), as could be expected, but also frequently on the fifth (CPC 7). For 4-note chords, there is tendency to start from the third (CPC 3 or 4) or the minor and major seventh (CPC 10 and 11) of a chord. It would be interesting to analyze the different harmonic uses of arpeggios in more detail, but this must be left for a future study.

Approaches

Approaches are typical for jazz line construction. Particularly the chromatic approaches, [−2, 1], [2, −1], [−1, 2], and [1, −2], are widespread. A closer look at the approaches as they are actually used is depicted in Table 4. Interestingly, with 17%, the most common approach is [−3, 1], a diatonic approach, most often targeting the root (CPC 0, 23%), the minor third (CPC 3, 18%) or the minor seventh of a chord (CPC 10, 16%). The chromatic approach [−2, 1] ranks second with 13.4%, most often targeting the major third (CPC 4, 19%), the root (CPC 0, 15%) or the fifth (CPC 7, 13%). The next nine ranks are occupied by diatonic approaches. The next to follow is [2, −1], the inverse approach to [−2, 1], with only 3.1% and with rather unusual targets (CPC 11 and 1).

Approaches show a clear trend towards targeting basic triadic chord tones (0, 3, 4, 7), making up 54% of all targets. If one takes major and minor sevenths (CPC 10, 11) and major sixths (CPC 9) into account, chord tones are targets of approaches 73% of the time. Generally, descending approaches are more common than ascending ones, and enclosures more frequent than escaping approaches.

The chromatic approach [−2, 1] targeting preferentially major triad chord tones, does indeed seem to be typical for jazz melodies, as it is the only highly ranked chromatic approach. A comparison with the Essen Folk Song Collections shows that there the approach [−3, 1] occurs about 100 times more often than [−2, 1], whereas in jazz these two are about equally frequent. We can only speculate as to the origin of this ›melodic habit‹. One possibility is that it might have originated in postponements or embellishments of chord tones, similar to a lower mordent.

Approach	Type	Most Common CPC Target	Rel. Freq. (%)
[–3, 1]	↓+°d	0, 3, 10	17.2
[–2, 1]	↓–°c	4, 0, 7	13.3
[–3, 2]	↓–°d	4, 9, 2	11.7
[3, –2]	↑–°d	0, 3, 7	10.8
[1, –3]	↓+^d	7, 0, 9	7.3
[–4, 2]	↓+°d	2, 0, 5	7.1
[3, –1]	↓+°d	4, 9, 2	6.9
[4, –2]	↑+°d	7, 0, 2	5.2
[2, –4]	↓+^d	3, 0, 10	4.8
[–1, 3]	↑+^d	5, 7, 2	3.9
[–2, 4]	↑+^d	2, 7, 4	3.3
[2, –1]	↑–°c	11, 1, 4	3.1
[2, –3]	↓+^d	4, 11, 9	2.2
[1, –2]	↓–^c	0, 10, 7	1.8
[–2, 3]	↑+^d	3, 10, 0, 6	0.8
[–1, 2]	↑–^c	6, 1	0.5

Table 4: Relative frequency of all 16 approaches. The column »Most Common CPC Target« indicates the three most common chordal pitch classes reached by this approach. Chordal pitch classes are pitch classes with values 0–11 in respect to the underlying chord. The overall direction is marked with ↓ for descending and ↑ for ascending; net movement of a semitone / whole tone is indicated with –/+; enclosures are marked with a degree ° sign, escaping approaches with a caret ^; intrinsically chromatic approaches are indicated with ›c‹, diatonic ones with ›d‹.

Links

In some regards, links (X atoms with one interval) are an artefact of the algorithm: a result of the decision in favor of non-overlapping segmentations. An overview of link values and relative frequencies can be found in Table 5. The most common is the descending minor third [–3], followed by the ascending semitone [1] and the ascending minor third [3]. In many cases, these links connect diatonic atoms with the same or different directions. In the case of ascending and descending minor thirds, this might be caused by pentatonic scales, which show patterns of whole tones and minor thirds. The ascending semitone most often links two descending diatonic atoms, typically as a small chromatic embellishment [–1, 1, –1] embedded in a descending diatonic line. This is partly due to the fact that diatonic atoms have priority over trills in the parsing algorithm. Here, a modification of the algorithm might be of interest. Whole tone links are likewise often produced as small embedded trills or by connecting two chromatic atoms.

Value	Rel. Freq. (%)
–3	14.3
1	10.9
3	10.8
–2	8.8
2	8.8
–4	6.8
0	6.7
5	5.7
–1	4.8
–5	4.3

Table 5: Values and relative frequencies of link atoms with relative frequency greater than 4%.

X atoms

X atoms were designed as a catch all category, initially hoped to be a rather small class. However, it turned out to be the second largest category or even the largest when combined with the link atoms. The large variability of the X atom class, however, makes a further sub-division of the category a rather complex endeavor, which must be deferred to follow-up studies. Here, only some preliminary insights and ideas are presented.

Value	N	Rel. Freq. (%)
[3, −3]	324	2.2
[−2, −3]	291	2.0
[−3, 3]	237	1.6
[2, 3]	221	1.5
[3, 2]	173	1.2
[−3, −2]	171	1.1
[1, 3]	164	1.1

Table 6: The most common X atoms with relative frequency greater than 1%.

In Table 6, the top seven values of X atoms with a relative frequency of over 1% are shown. The most common X atom value is a minor third cambiata [3, −3], which only, however, accounts for 2.2% of all X atoms. The top six values contain only minor thirds and major seconds, which is evidence that pentatonic scales might deserve their own category. The seventh most common X value, [1, 3], seems to be an artefact of the transcription process used for the WJD or might be a result of blues intonation, since the most common start chordal pitch class is the blues third (23%), which results in a pitch sequence of a blues third, a major third and a fifth over a chord, all of which are part of the major blues scale. Searching for and listening to instances of this interval pattern using the Dig That Lick Pattern Search web application[3] (Frieler et al. 2018) corroborates this fact, since in many examples the 1 interval is actually an appoggiatura or acciaccatura.

In Fig. 6, some typical examples of longer X atoms are displayed. Fig. 6a shows an interwoven line, which is already a sub-category of the *line* MLU category. Interwoven lines consist of two lines (in all possible combinations of directions, including a repetitive one), which are played in an alternating fashion, often separated by a large interval, which results in a kind of pseudo-polyphony. It would be desirable, but challenging, to devise a parsing algorithm for these kinds of lines.

3 https://dig-that-lick.hfm-weimar.de/pattern_search/search.

Fig. 6b is a rather abstract sequence of tones and intervals [−2, −4, 5, 2, −3, 4, −3, 6, −5, 5], with no apparent structure and tonally very ambiguous. Fig. 6c displays an expressive tone sequence, played in the highest register of the tenor saxophone, with a small overall ambitus of a tritone, an irregular interval sequence and rhythm, and oscillating between two central pitches a minor third apart. The overall impression can be compared to ›screaming‹ or ›howling‹. Fig. 6d shows a combination of a descending Db-major pentatonic line and an ascending G minor pentatonic line. Fig. 6e is an F minor pentatonic lick, with a bluesy impression. Finally, in Fig. 6f, a minor third trill is displayed.

These six samples give only some preliminary insights into the diverse world of X atoms, where interesting licks and motifs can be found. To this end, the WBA grammar allows the user to filter for ›peculiarity‹, since diatonic, chromatic, and arpeggio atoms are very common and thus melodically of lesser interest.

WBA grammar and Markov chains

In this section, we will investigate first order Markov chains of WBA atoms. As we recall, a sequence of events is said to fulfill the Markov property (of Nth order) if the probability of observing an event e_i is only dependent on the preceding N elements in the chain, formally, $p(e_i | e_{i-1} \ldots e_1) = p(e_i | e_{i-1} \ldots e_{i-N})$, or $p(e_i | e_{i-1} \ldots e_1) = p(e_i | e_{i-1})$ for first order models. This must be read as the probability of observing event e_i after having observed the previous event e_{i-1}. If the probability of observing an event is independent of the events before it, e.g., when throwing a dice twice, then we have a Markov model of zeroth order. This mean that the process is memory-less, and that events follow each other in purely random fashion. The question now is: which is the best Markov for sequences of WBA atoms (neglecting length and direction)?

Since proving the Markov property for empirical data is a non-trivial task, we will restrict ourselves here to examining the first order Markov transition matrix for WBA atoms for *line* and non-*line* MLUs separately. The transition matrix for the WBA segmentation is the matrix p_{ij} of probabilities $p(x_i | x_j)$ for all elements $x_i \in \{A, C, D, F, L, J, R, T, X\}$. These are the probabilities that after observing atom x_j, the atom x_i will follow. They can be estimated empirically by the relative frequencies of all bigrams $e_i e_{i-1}$ in our data. To test the Markov properties, the transition probabilities can be compared

a
b
c
d
e
f

Fig. 6: Examples of X atom types. (a) consists of two interwoven descending lines (pseudo-polyphony), (b) abstract atonal line, (c) expressive, asymmetric trill, (d) double pentatonic arpeggio/line, (e) bluesy, pentatonic lick, (f) minor third trill. Solo sources: (a) Roy Eldridge, »Undecided«; (b) Steve Coleman, »Processional«; (c) Joe Lovano, »Lonnie's Lament«; (d) Bob Berg, »No Moe«; (e) Woody Shaw, »Rosewood«; (f) Branford Marsalis, »Housed from Edward«.

to the zero order probabilities $p_i = p(x_i)$. If the atoms are following each other independently, the transition probabilities should then satisfy $p_{ij} = p_i$, $p_{ij}/p_i = 1$, or $\log(p_{ij}/p_i) = 0$, which is the point-wise self-information (PSI). Hence, if we know the confidence intervals (CI) for the PSI value, we can test whether these enclose zero or not. If a CI excludes zero, this is empirical evidence that the atoms do not succeed each other independently and that there is a causal link between them. Since the a priori distribution of the PSI values is not known, we resort to a bootstrap procedure (Efron 1979) to estimate confidence intervals from the data. This is done by randomly sampling with replacement from MLUs of interest while calculating zero and first order transition probabilities from this sample. This yields empirical distributions for the probabilities from which the 5% and 95% quantiles can be estimated, which gives the 95% confidence interval. To this end, we choose 100 batches of samples with a size of 1% of the total number of relevant MLUs. For instance, there are 4,609 *line* MLUs in the WJD, so we choose 100 bootstrap samples of 46 *line* MLUs each.

The results for *line* MLUs and all other MLUs are displayed in Fig. 7. The top row shows the first order transition probabilities, the bottom row the PSI values, with indication of significant deviation from independence. A first observation is that the transition matrices for *line* and non-*line* MLUs are qualitatively rather similar, with the main differences stemming mostly from the different composition of WBA atoms (cf. Fig. 2), with diatonic atoms more and repetition and trill atoms less frequent in *line* MLUs. However, as can be seen from the PSI values, there are much more significant first order transitions for the *line* MLUs. A large part of these come from repetition atoms alone, which are rare in general and even more so in *line* MLUs, so the effect of this is hardly relevant. The diminished self-transitions of A, D, and C atoms is explainable by the fact that these imply a change of direction, e.g., an ascending arpeggio immediately followed by a descending one, hence the number of possibilities is roughly halved. Performers seem instead to prefer inserting a link before continuing, as can be inferred from the heightened transition probabilities from links and X atoms to diatonic and chromatic atoms for *line* MLUs. Interestingly, however, the same cannot be observed for the non-*line* MLUs. The increase of transitions to links and X atoms is most likely by construction, as these are the ›catch-all‹ category, and self-transitions between them are not possible. Likewise, the non-*line* MLUs show nearly no significant transitions, except for X and L atoms, so it

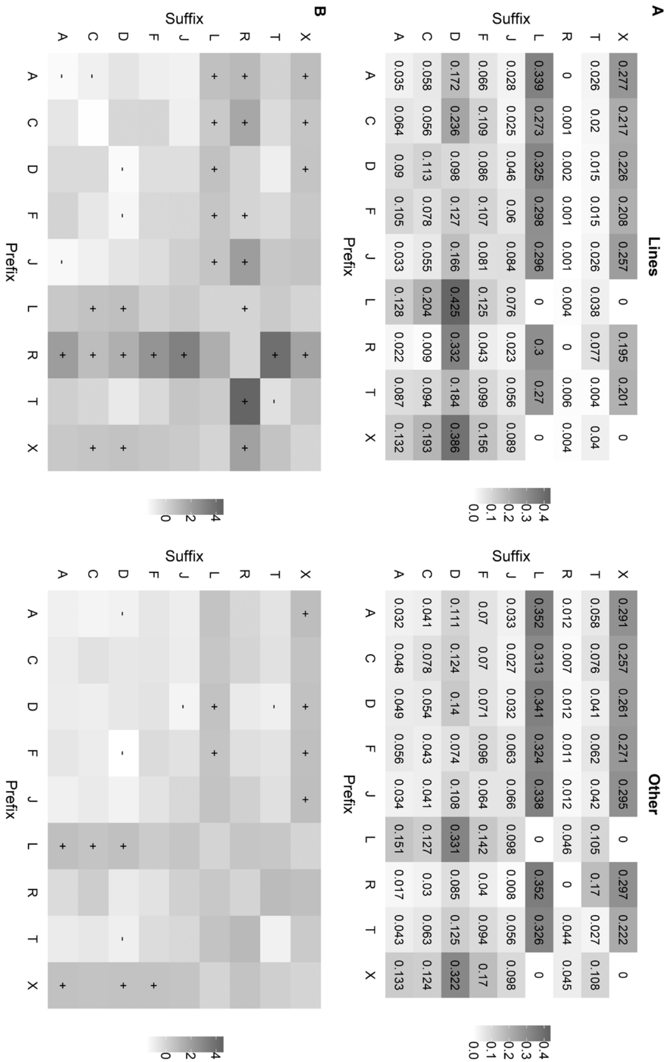


Fig. 7: (A) Markov transition probability matrices for line and other MLUs. Note that some transitions are impossible by construction, i.e., R ⟶ R, L ⟶ L, L ⟶ X, X ⟶ L, and X ⟶ X. (B) Pointwise self-information matrices, based on 100 bootstraps samples each. Transitions significantly different at the α = .05 level are marked with ›–‹ for a lower probability and ›+‹ for a higher probability than the zero order probabilities. PSI is measured in bits, so a value of 1 corresponds to a twice larger probability, a value of 2 to a four times larger probability etc.

appears that WBA atoms are basically independently (randomly) chained here. In conjunction with the fact that the significant transitions for *lines* are also either trivial or mostly due to specificities of the parsing algorithm, it can be stated that a zeroth order Markov model is appropriate. Hence, the grammar for WBA atoms has no deep structure.

Discussion

We proposed a novel representation of melodies based on a system of melodic atoms, the Weimar Bebop Alphabet. It was devised with the performer's perspective in mind as an intermediate level of abstraction. We were able to demonstrate that different midlevel units have slightly different WBA atom distributions, where the L, D, and X atoms always dominate. On the one hand, this reveals the diatonic roots of jazz melodies but on the other hand, it also calls for further refinement of the system.

X atoms turned out to be second largest category of all. This reflects the high variability and also the melodic inventiveness of the jazz performers, but runs counter to the original assumption that it would be possible to achieve a considerably simplified (more structured) description of jazz solos with the help of WBA. Still, some simplification is provided by the WBA system in facilitating a classification of melodic chunks, which proved to be a helpful analytical tool, particularly for separating the melodic ›wheat‹ from the ›chaff‹. However, the complexity of melody description does not seem to be substantially reduced. Since the first order transition matrices are largely identical to the zero order probabilities, no true grammatical structure can be found at this level of abstraction. Further study is necessary to disentangle whether this is simply a shortcoming of the approach or further evidence of the intrinsic complexity of jazz improvisation. Some of the observed results might be due to the lack of attention paid to the harmonies (or tonality in general) underlying the lines, since only interval sequences are considered here. However, basing the WBA system on intervals is the easiest way to achieve transposition invariance and is also universal in the sense that it can be applied equally to melodies with and without harmonic accompaniment.

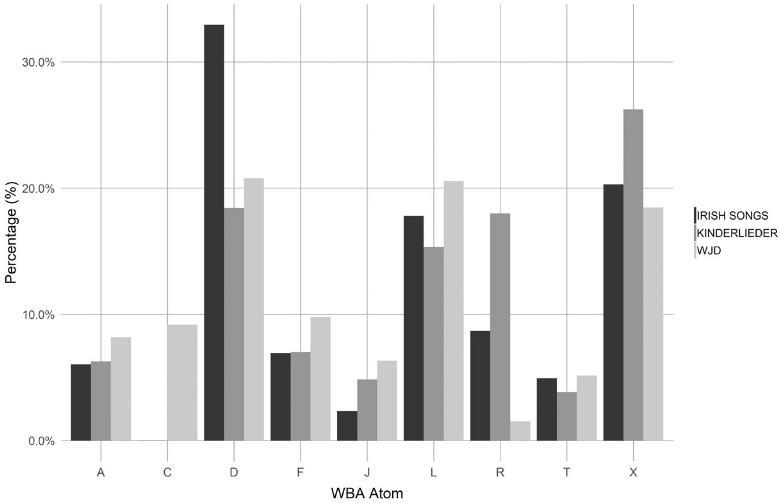


Fig. 8: Comparison of distribution of WBA atoms in the WJD and two sub-collections from the Essen Folksong Collection: IRISH songs (53 tunes) and KINDERLIEDER (German children songs with 217 tunes).

In regard to the universality of the system, applying the WBA algorithm to two collections (IRISH songs and KINDERLIEDER) from the Essen Song Collection (Schaffrath 1995) provides some insights (Fig. 8). The main differences between the WJD and those collections are the complete absence of chromatic atoms from the folk songs, the much smaller percentage of repetitions in the WJD, and the much larger share of diatonic atoms in the Irish songs. In all other aspects, the distributions are comparable. Further inspections, however, show that, for example, the X and F atoms in the WJD are rather different in content as compared to the folk songs. These differences might be explored in the future. Right now, it suffices to say that the WBA is a universal melody description system, and might well be renamed Universal Melody Alphabet (UMA).

Fig. 9: Automatically generated chorus of an F blues solo using a hierarchical first order Markov model of MLUs and WBA.

Conclusion and outlook

In the future, the system may undergo further refinements, for example, by trying to subdivide the X atom category (see sect. 4.2.4). Interwoven lines and other symmetrical patterns seem common, but their diversity and frequent slight variations make developing a parser a challenging task. Furthermore, trills might be expanded to comprise larger intervals, as third trills, for example, seem to be rather frequent.

Of course, the analytic potentials of system are not yet fully exhausted. Several avenues for further research have already been suggested, including comparative studies, either with different corpora, such as the Essen Folk Song Collection, but also between different performers or styles.

Finally, the author has already informally explored another possibility: The WBA system could be used for the automatic generation of monophonic jazz solos. Generated solos are helpful tools for evaluating and refining models of jazz improvisation via an iterative analysis-by-synthesis process.

According to this approach, a hierarchical model with a first order Markov model of MLU at the top level generates a plausible global phrase structure. Next, the MLUs are realized via first order Markov models of WBA atoms with different models for different MLUs. The sequences of WBA atoms can then be realized with tones according to a given sequence of chords, while inserting ad-hoc links to reach chord tones if needed. The advantage of using this level of abstraction in comparison to, e.g., note-level Markov models, is that overall shapes and typical traits of jazz solos can be achieved more easily as they produce short chunks of abstract descriptions which can be easily adapted to the underlying harmony, e.g., using a mixolydian scale to realize a descending diatonic atom over a dominant seventh chord. An example of one chorus over an F blues chord sequence is depicted in Fig. 9.

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